Geography in Coverage Modeling: Exploiting Spatial Structure to Address Complementary Partial Service of Areas

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Abstract

The assessment of, and planning for, service coverage has been a fundamental aspect of geographic research. In particular, facility placement and associated coverage are central concerns in emergency services, transit route design, cartographic simplification, natural resource management, and weather monitoring among others. In this paper the widely applied set covering problem is discussed, focusing on its use in geographic analysis. Problematic aspects of set coverage modeling across space are identified. In particular, geographic information systems (GIS) and enhanced spatial information have accentuated abstraction/spatial representation issues in need of greater consideration in modeling service coverage. To address representational problems with existing approaches, a new set covering model is introduced for dealing spatial objects (point, lines, polygons, arcs, curves, etc.). The developed approach accounts for complementary coverage of objects. In doing this, the model decreases modifiable areal unit problem impacts known to be an issue in the geographic application of the set covering problem. Empirical results are presented to support the usefulness and validity of this new approach.
Introduction

Coverage is a concept that has been broadly interpreted in quantitative modeling. One of the classic areas that covering has been utilized is in crew scheduling (aviation, rail, etc.), where scheduled trips must be “covered” by a crew at minimum total cost (see Rubin 1973; Caprara, Toth, and Fischetti 2000). From a management perspective the idea is to have the fewest number of crews (or least cost) possible in order to fulfill or cover scheduled operations. Related applications have been noted for truck routing and political districting as well as circuit design (see Roth 1969; Balas and Padberg 1972). Edmonds (1962) was among the first to detail what is now referred to as the set covering problem (SCP), where one is interested in the set of servers (crews, facilities, etc.) of minimum total cost required to cover needs (scheduled routes, user demand, etc.).

In geography coverage is a prominent term as well. A fundamental tenet in central place theory is range (Christaller 1966). In the context of central place, range reflects the maximum distance or time a patron is willing to travel if a service center is to be used (Hurst 1972). Range has been interpreted in spatial analysis as a coverage standard, indicating the sub-region that could be served by a particular facility. Toregas et al. (1971) sought to identify a minimum number of fire stations to provide suitable service to a region using the SCP, where suitable service corresponded to a coverage standard ensuring response time within a pre-specified limit. Related geographic applications of the SCP include express bus stop placement (Gleason 1975), fire station location (Plane and Hendrick 1975), line generalization (Cromley and Morse 1988), warning siren siting
(Current and O'Kelly 1992), nature reserve design (Ando et al. 1998), and weather radar station location (Minciardi, Sacile, and Siccardi 2003) among others.

The SCP continues to be useful for addressing a variety of planning problems, which should be no surprise given the breadth of applications noted thus far. Two general lines of work are found in the SCP literature. The first is that new or different application areas are identified where the SCP has proven, or could prove, beneficial. Examples include species preservation (Ando et al. 1998), and weather monitoring (Minciardi, Sacile, and Siccardi 2003). The second is a focus on developing improved solution techniques for the SCP. That is, the SCP remains a challenging problem to solve, particularly for large or unique problem instances, so specialized and improved approaches continue to be needed (see Beasley 1990; Ceria, Nobili, and Sassano 1997; Caprara, Toth, and Fischetti 2000; Brotcorne, Laporte, and Semet 2002).

This paper deviates somewhat from current efforts by exploring the SCP with respect to its use in a geographic context. Recent work by Murray and O’Kelly (2002) and Murray, O’Kelly, and Church (forthcoming) suggests that there are scale and unit definition issues, the so-called modifiable areal unit problem (MAUP), when the SCP is applied in spatial studies. The MAUP suggests that a quantitative measure or model may be susceptible to manipulation by altering spatial scale (i.e., Census blocks versus tracts for a given region) or by changing how spatial units are defined (i.e., postal code boundary versus policing districts, where the same number of units are used but the boundaries differ). Given that many quantitative methods have been found to be subject to MAUP
(see Openshaw and Taylor 1981; Fotheringham and Wong 1991; Murray and Weintraub 2002), observed variability in the SCP could be anticipated. It is unfortunate that little has been done thus far to reflect on the ways in which spatial representation issues may be critical when using the SCP, or other location models for that matter. This is all the more significant when considering the call by Tobler (1989) for frame independent quantitative measures. Specifically, Tobler (1989) suggests that any method of analysis that is subject to manipulation based upon a change of scale or unit definition may be inappropriate. If an approach is subject to MAUP, the challenge Tobler (1989) raises is to explore whether a frame independent alternative exists.

In this paper we revisit set coverage and its use in geographic analysis. The next section details the set covering problem. This is followed by a discussion highlighting problematic aspects of set coverage in spatial contexts. In particular, we note how geographic information systems (GIS) have changed how coverage may be approached, challenging researchers to confront spatial abstraction. To address representational issues, a new set covering model is proposed. Application results are presented to support the usefulness and validity of this new approach. Finally, a discussion and concluding comments are given.

**Set Covering**

The set covering problem emanates from set theory in mathematics, where one is interested in the smallest subset of potential facilities, as an example, that provides
suitable coverage to all demand areas. In order to formally state the SCP in the context of facility location, consider the following notation:

\[ i = \text{index of demand areas;} \]

\[ j = \text{index of potential facility sites;} \]

\[ f_j = \text{fixed cost to locate at potential site } j; \]

\[ a_{ij} = 1 \text{ if area } i \text{ is suitably served by a potential facility at site } j, \text{ and } 0 \text{ otherwise;} \]

\[ x_j = \begin{cases} 
1 & \text{if potential facility site } j \text{ is selected for service establishment} \\
0 & \text{otherwise.} 
\end{cases} \]

Given this notation, \( a_{ij} \) reflects which potential facility locations provide suitable coverage to particular areas of demand. Thus, \( a_{ij} \) is determined/derived prior to its use in an optimization model. One example using \( a_{ij} \) is fire department response time, where suitable service is considered on site arrival of equipment in 8 minutes or less once a call has been received (see Green 1994). Thus, \( a_{ij} = 1 \) if area \( i \) can be reached in 8 minutes from facility \( j \), or \( a_{ij} = 0 \) if it cannot. Another example is emergency warning siren coverage, where \( a_{ij} = 1 \) if area \( i \) is within the audible sound region of a siren located at site \( j \), or \( a_{ij} = 0 \) otherwise (Current and O’Kelly 1992). It should be evident then that there is no assumed regularity of shape for a coverage region emanating from site \( j \), as a given region may be regular, irregular, contiguous or non-contiguous depending upon how service is provided.
With the above notation, the basic set covering problem can be detailed:

**Set Covering Problem (SCP)**

\[
\text{Minimize } \sum_j f_j x_j \tag{1}
\]

Subject to

\[
\sum_j a_{ij} x_j \geq 1 \quad \forall i \tag{2}
\]

\[
x_j = \{0,1\} \quad \forall j \tag{3}
\]

The objective, (1), of the SCP is to minimize the total weighted facility cost in providing service. Constraints (2) require suitable coverage of each service area by at least one sited facility. Constraints (3) impose integer restrictions on decision variables.

Of course the importance of the SCP is its use for addressing a planning problem. Thus, obtaining a solution for a particular application instance is essential. As noted previously, considerable effort has been devoted to the development of approaches for solving the SCP using exact and approximate (or heuristic) methods. A common approach for solving the SCP exactly has been integer programming (Toregas et al. 1971; Current and O’Kelly 1992; Ceria, Nobili, and Sassano 1997; Minciardi, Sacile, and Siccardi 2003). An exact method ensures that the best (or lowest total cost in this case) solution is found. However, for various reasons heuristic solution approaches continue to be in demand. Heuristics characteristically are specialized, problem specific techniques that find good feasible solutions quickly. Reasons for relying on a heuristic include cost (commercial
optimization software can be expensive), problem size (encountered problems are too
large for commercial optimization software to solve), performance expectations
(commercial optimization software is too slow in deriving an optimal solution) or
problem characteristics (application instances produce a structure that simply cannot be
solved by an exact method). Heuristics that have been developed and employed for the
SCP are reductions, Lagrangian relaxation, greedy and a range of meta-heuristics (see
Roth 1969; Toregas and ReVelle 1972; Beasley 1990; Ceria, Nobili, and Sassano 1997).

Set Coverage Issues

As noted previously, set covering has been of theoretical and practical interest for some
time. In the geographic domain, early use of the SCP relied on points for representing
demand areas and points for representing potential facility sites. This has largely
continued to be the case. Such an abstraction has been useful for many reasons, but no
doubt the evaluation of coverage is facilitated by the use of points because distance can
readily be assessed (e.g. Euclidean or network shortest path distance between two points).
Another practical issue with the SCP has been that problem size was rather limited due to
the computational demands of exact approaches, as Church (2002) notes.

Geographic information systems (GIS) have changed, and are continuing to change, how
spatial analysis in general is approached (O’Sullivan and Unwin 2003). One aspect of this
change is significant advances in computational processing capabilities. Related to the
SCP, Miller (1996) discusses ways in which GIS can expand how location planning
models are conceived, moving beyond a simplistic point-based representation of spatial
entities being modeled. Church (2002) provides a functional review of GIS in relation to location modeling, discussing how GIS provides capabilities for: integrating multiple data layers into a common coordinate system; manipulating information in order to change spatial scale or aggregate (or disaggregate) spatial units; buffering (using distance or travel time) objects and spatial query; and, visualizing data and model results.

There is little doubt that GIS facilitates access to geographic information, typically at a variety of spatial resolutions with numerous, if not hundreds or thousands of, associated attributes. Given this, GIS affords considerable flexibility in how an SCP could be represented (or abstracted). For example, a siting study could opt to use one of a number of Census levels (e.g. block, block group, tract, etc.), each corresponding to a different spatial scale. Further, using GIS one can readily modify a particular geographic representation. For example, it is rather simple to extract centroids of blocks, if points are desired, or engage in a process of defining new spatial entities for a region of interest. Both cases are illustrated in Figure 1, where block centroids are shown in Figure 1a and regular tessellations are given in Figure 1b. Thus, Figure 1 shows three potential ways for representing the same region -- blocks, block centroids or regular tessellations. It is conceivable that any of these three (or others) could be utilized in a location model, particularly the SCP.

From a functional perspective, a major feature of GIS is the ability to carry out interactive analysis using a variety of querying approaches, non-spatial and spatial (O’Sullivan and Unwin 2003). For the SCP, GIS enables geographic coverage to be assessed, both simple
and complex in design. For example, Figure 2 identifies the emergency warning coverage region of a siren under two scenarios. The “Theoretical coverage buffer” layer shows the expected sound transmission radius of 5200 ft. corresponding to the 70 dB or higher range. This assumes no sound obstructions. In contrast, the “Actual siren coverage” layer in Figure 2 depicts a more probable coverage region, taking into account neighboring buildings that deflect and/or hinder sound propagation. Of course in most vector GIS software one can query (or buffer in this case) points, lines or polygons in various ways, taking into account other information layers, travel time, orientation, and/or direction (see Farhan and Murray, forthcoming). Thus, one can derive a spatially appropriate coverage region for the type of service being provided using GIS. As a result, a coverage region may be regular (i.e., circle, ellipse, square, rectangle, etc.), irregular (i.e., polygon) or non-contiguous in shape. A final feature of GIS to be mentioned here is its use in visualizing spatial information, as Church (2002) highlights. Not only can a user explore potential patterns in geographic information and associated attributes, but with respect to the SCP it is possible to examine a proposed siting configuration.

Visualization was an important aspect of recognizing scale and unit definition issues (MAUP) in the application of the SCP by Murray and O’Kelly (2002) and Murray, O’Kelly, and Church (forthcoming). For a particular region (the City of Dublin shown in Figure 1) and a specific facility type (a warning siren having a service coverage radius of 3200 ft.) with each potential site having the same relative cost, Murray and O’Kelly (2002) found that the minimum number of necessary facilities identified using the SCP

1 These characteristics are for the Whelen WPS 4000-4 outdoor emergency warning siren (www.safetycom.com). The 70 dB level is considered the minimum suitable sound level by this industry and associated planning agencies.
varied significantly depending on the underlying point representation employed. Specifically, they found SCP results ranging from 14 to 27 facilities for the provision of coverage at minimum total cost. Further, while each SCP solution did cover all underlying points utilized in its associated representation, coverage of the actual region abstracted by the points was never achieved in any of the 51 representations examined. Murray and O’Kelly (2002) reported coverage gaps ranging from 0.41% to over 27% of the total area. Beyond this, the behavior of the SCP did not conform to expectations as point density increased. That is, as point density increases uniformly, one would expect fewer coverage gaps of the actual region. This did not happen. However, computational effort did increase substantially as the number of points to be covered increased, but this is an expected property of the SCP. In summary, Murray and O’Kelly (2002) found that not only do coverage gaps result when applying the SCP to a point-based representation of a region, but also that even for relatively small total coverage gaps the number of identified facilities can be nearly 30% lower than actually needed.

Given the goal of providing coverage to the actual region that points typically serve as an abstraction of, Murray, O’Kelly, and Church (forthcoming) explored using polygons (or tessellations) as demand areas to be fully covered. Murray, O’Kelly, and Church (forthcoming) demonstrated that GIS could readily assess coverage of polygons (or any spatial object), and with this an associated SCP could be structured. The benefit of covering areas as opposed to points is that total coverage of the region can be guaranteed using the SCP, because each individual areal unit is required to be completely covered by at least one service facility. Murray, O’Kelly, and Church (forthcoming) found that
applying the SCP to different spatial representations of polygons (like those shown in Figure 1b) produced inconsistent results as well. Also examining the region shown in Figure 1 using the same type of facility, Murray, O’Kelly, and Church (forthcoming) found that the SCP identified between 26 and 57 facilities as being needed. There was essentially no agreement in the 16 different representations analyzed, but as polygon size decreased the number of needed facilities generally decreased (to a lower limit of course). Similar to Murray and O’Kelly (2002), as problem size increased, Murray, O’Kelly, and Church (forthcoming) reported that computational difficulty for solving the associated SCP increased substantially as well.

The findings of Murray and O’Kelly (2002) and Murray, O’Kelly, and Church (forthcoming) confirm the existence of the MAUP in the application of the SCP. This is not surprising given that many spatial analytical methods are subject to the MAUP too (see Openshaw and Taylor 1981; Fotheringham and Wong 1991; Murray and Weintraub 2002). What remains is how to deal with MAUP issues in the context of the SCP. In most location studies, researchers suggest using the most disaggregate data available (Horner and Murray 2002), implying that this is the best that can be done. However, given the challenge for seeking out frame independent quantitative methods issued by Tobler (1989), it is important to reconsider aspects of set coverage in order to assess the potential for an alternative modeling approach to this basic problem.

**Revisiting Set Coverage**
One common assumption in the spatial application of the SCP is that complete regional coverage is provided. Given this, analysis relying on point-based representations of demand, where coverage gaps remain, may therefore be problematic. Murray, O’Kelly, and Church (forthcoming) demonstrated that the use of areal units for representing geographic space ensures that no coverage gaps result. However, an excessive number (or total cost) of identified facilities may result. Murray, O’Kelly, and Church (forthcoming) point out that the binary interpretation of polygon coverage/non-coverage necessarily underestimates actual service afforded by a SCP facility configuration, with larger average polygon size contributing to greater underestimates of provided coverage. As a result, a greater total cost for facilities or services is indicated as being necessary using the SCP. Such a finding, resulting from different spatial representations of the same region, is indicative of the MAUP. Addressing MAUP issues when areal units are used in the SCP likely hinges on reducing or eliminating representational underestimation of actual coverage. Of course one way to deal with this is along the lines of the conventional wisdom noted above, use the most disaggregate data possible or further disaggregate acquired areas into even smaller spatial units. Unfortunately this does not address the spirit of the MAUP, plus computational issues are quickly encountered thereby limiting the size of problems that can reasonably be analyzed in practice (see Murray, O’Kelly, and Church, forthcoming). GIS provides an alternative means for addressing the underestimation associated with the use of areal units.

Not only can one readily undertake spatial query and object buffering operations in GIS, but various computational geometry functions are available as well. Specifically, it is
possible to assess containment and overlap. For example, the red polygons (1, 4, 7 and
10) in Figure 3 are fully contained in the siren A service area delineated by the red circle.
Further, polygons 2, 5, 8 and 11 overlap with the coverage area of siren A. That is, they
are only partially within the service area of siren A. It is precisely these overlapping
polygons that represent an underestimate of actual coverage using the SCP. The reason
for this is that from a modeling perspective only those polygons fully covered would be
accounted for in the SCP (e.g. \( a_{1A} = 1 \), \( a_{4A} = 1 \), \( a_{7A} = 1 \), \( a_{10A} = 1 \), and \( a_{iA} = 0 \) for \( i=2, 3, 5, 6, 8, 9, 11, 12 \)) as the desired outcome of the model is complete regional coverage.
Clearly omitted here is the fact that some polygons are partially covered (e.g. 2, 5, 8 and
11 in Figure 3). In fact, Figure 3 shows a case where the combined partial coverage of
polygons 5 and 8 by sirens A and B actually results in complete coverage of these
polygons. Accounting for this situation in the SCP may well begin to address observed
MAUP issues. Using GIS, such overlap or partial coverage can be assessed. As a result, it
is possible to account for full and partial coverage of spatial objects (i.e., lines, polylines,
polygons, circles, ellipses, etc.) by potential facilities. The challenge that remains is
incorporating partial coverage considerations in an optimization framework so as to
ensure complete coverage of the region of interest.

Consider the following additional notation:

\[ \beta = \text{minimum acceptable coverage percentage in the range } [0, 100] \]
\[ \Omega_i = \text{set of potential facilities } j \text{ partially covering area } i \text{ at least } \beta, \text{ but less than } 100\%; \]
\[ \alpha = \text{minimum number of partial coverage facilities needed for complete coverage}; \]
\[
y_i = \begin{cases} 
1 & \text{if area } i \text{ is partially covered at least } \alpha \text{ times} \\
0 & \text{otherwise.}
\end{cases}
\]

Here \( \beta \) represents a pre-specified threshold on partial coverage. That is, we are interested in identifying those areas that are at least \( \beta \% \) partially covered. For example, in Figure 3 we could establish \( \beta = 50\% \). With this we can determine the associated partial coverage set for each area, those potential facilities that partially cover each area \( i \). For Figure 3, this would mean that \( \Omega_2 = \{A, B\} \), \( \Omega_3 = \{A, B\} \), and \( \Omega_i = \{\emptyset\} \) for \( i = 1, 2, 3, 4, 6, 7, 9, 10, 11, 12 \) when \( \beta = 50\% \). From a coverage modeling perspective, if we observe that an area is partially covered multiple times and this sum is greater than or equal to 100\% of the area, then it is highly probable that the area is in fact completely covered. Thus, \( \alpha \) establishes this multiple partial coverage of an area and would necessarily be linked to the stipulated coverage fraction \( \beta \). As an example, in Figure 3 \( \beta = 50\% \) and \( \alpha = 2 \) represents this linkage between these two parameters for addressing partial coverage by facilities. Though the focus here is oriented toward covering areas, any spatial feature (lines, polylines, polygons, circles, ellipses, arcs, etc.) would theoretically be applicable as well. The decision variables \( y_i \) track which areas are partially covered at or above this minimum level. This notation allows us to introduce a new version of the set covering problem.

\[
Set\Covering\Problem\Spatial\Objects\(\text{(SCP-SO)}\)
\min \sum_j f_j x_j \quad (4)
\]
Subject to

\[ \sum_j a_j x_j \geq 1 - y_i \quad \forall i \] 

(5)

\[ \sum_{j \in \Omega_i} x_j \geq \alpha y_i \quad \forall i \] 

(6)

\[ x_j = \{0,1\} \quad \forall j \] 

(7)

\[ y_i = \{0,1\} \quad \forall i \] 

The SO extension reflects any spatial object (point, line, arc, polygon, circle, etc.) as a possible demand entity requiring coverage. The objective, (4), of the SCP-SO is to minimize the total weighted facility cost in providing service, and is unchanged from (1). The change in SCP-SO is incorporating the possibility of complete partial coverage, and is accomplished using Constraints (5) and (6). Constraints (5) require suitable coverage of each service area by at least one sited facility if \( y_i = 0 \). However, if \( y_i = 1 \) in (5), then no constraining condition is imposed. Constraints (6) require areas partially covered \( (y_i = 1) \) to have at least \( \alpha \) facilities providing this partial coverage. If \( y_i = 0 \) in (6), then no constraints are enforced in the SCP-SO. Constraints (7) impose integer restrictions on decision variables.

In the SCP-SO Constraints (5) and (6) track full and partial coverage of demand areas, ensuring that each area is either completely covered by at least one sited facility or partially covered by a minimum number of sited facilities. The decisions to be made in the SCP-SO are where to locate facilities and which areas will have either full coverage or an established threshold of partial coverage.
There is no attempt to track exactly which facility is providing partial coverage in the SCP-SO. Consequently, there is a possibility that partial coverage by multiple facilities would not be complementary, and as a result complete regional coverage not achieved. As an example, assume $\beta=50\%$ and $\alpha=2$ for the configuration shown in Figure 4. Polygon 8 in this case is partially covered, with $\beta\geq50\%$, by both siren A and siren B, yet complete coverage is not actually provided. However, the objective of the SCP-SO, and the SCP, is to minimize total cost in order to provide complete coverage. This means that facilities will seek to disperse spatially to the greatest extent possible, thereby making complementary coverage essential and highly probable. Accordingly, the SCP-SO intentionally avoids the requirement of enumerating all possible configurations of partial coverage given this *fundamental insight* about the properties of set covering in geographic space.

One interesting feature of the SCP-SO is that for $\beta=100\%$ and $\alpha=1$ there is an equivalence to the SCP. Thus, the SCP is a special case of the SCP-SO. This is expected conceptually given that the SCP-SO relaxes the strict full coverage of a demand area imposed in the SCP.

Another feature of the SCP-SO, as well as the SCP, worth noting is that no feasible solution may exist for some service standards. Consider the following set:

$$N_i = \{ j \mid a_{ij} = 1 \}$$
Thus, $N_j$ represents the set of potential facilities $j$ that can suitably cover area $i$. If $|N_j| = \emptyset$ and $|\Omega_i| < \alpha$ for any $i$, then no siting configuration will be capable of completely (or sufficiently) covering area $i$.

**Application**

The City of Dublin, Ohio is used in this research for comparative purposes as it has been the subject of previous analysis (see Current and O’Kelly 1992; Murray and O’Kelly 2002; Murray, O’Kelly, and Church, forthcoming). For this region, the SCP-SO is applied to site a minimum number of emergency warning sirens in order to impart regional coverage. Service is provided by omni-directional sirens (Whelen WPS-2750) that have a rated maximum effective range of 3200 ft. at the 70 dB level, and cost approximately $13,000 per siren excluding installation. Consequently, a system of warning sirens can be a considerable budgetary concern for a relatively small municipality like Dublin (less than 40,000 residents), so quantitative analysis oriented toward the most efficient placement of these services is very much in demand.

Discrete potential facility locations were established such that a siren could be located every 328 ft. in the north/south or east/west direction within the City of Dublin. This resulted in 4,586 potential facility sites. The spatial pattern is similar to the tessellation vertices shown in Figure 1b, with distances between points (or vertices) being 328 ft. Similarity in fixed costs to site at the potential facility locations in this region is assumed,
common in location coverage studies (see Toregas and ReVelle 1972). Thus, \( f_j = 1 \) for all \( j \) in the analysis reported here. In addition, the service coverage area for each potential facility is the same, a circle with a radius of 3200 ft. These assumed problem characteristics are consistent with those established by Murray and O’Kelly (2002) and Murray, O’Kelly, and Church (forthcoming), enabling comparison of results. The analysis conducted by Murray and O’Kelly (2002) and Murray, O’Kelly, and Church (forthcoming) for the Dublin region identified between 14 and 57 sirens as being necessary. At a minimum cost of $13,000 per siren, this translates to a total system cost of at least $182,000 to $741,000.

The application of the SCP-SO was carried out on a Pentium III 1 GHz personal computer running Windows NT (ver. 4.0) with 524 MB RAM. ArcView was utilized to manage geographic data for this region. An ArcView interface was programmed using Avenue to structure the SCP-SO for siting sirens in the Dublin region. ArcView was fundamental for deriving and evaluating service coverage, enabling associated optimization problems to be written to a text file. Specifically, ArcView is used to determine which potential facility locations \( j \) fully and partially cover each area \( i \), sets \( N_i \) and \( \Omega_i \) respectively, given the coverage standard and parameters \( \alpha \) and \( \beta \). With this information, the formal mathematical program can be structured. Problems then are solved externally to ArcView using CPLEX. Once the problems are solved, the solution is then exported from CPLEX and read into ArcView for display and further analysis.
The SCP-SO was first applied to determine a facility configuration providing coverage to the 153 Census blocks shown in Figure 1a. The first case to be examined is for $\beta=100\%$ and $\alpha=1$, which of course is equivalent to the SCP, requiring full coverage of each polygon (or block in this case). There is no feasible solution to this problem because it is not possible to cover all blocks using only one siren. That is, there are one or more polygons that are too big relative to individual service provided by a siren ($|N_i| = \emptyset$ for one or more areas $i$), so no valid (or feasible) solution can be found for $\beta=100\%$ and $\alpha=1$.

The second case is for $\beta=50\%$ and $\alpha=2$. The SCP-SO identified 27 sirens as being necessary for covering the entire region. This configuration is shown in Figure 5. Computationally, only 6.66 seconds (5516 iterations and 92 branches\(^2\)) were needed using CPLEX to solve this problem optimally.

There are two aspects of the SCP-SO solution shown in Figure 5 worth further discussion. First, complete regional coverage is not actually achieved. In this case, 5.84\% of the region is not suitably covered by siren service. Whether this is a significant total coverage gap is open to interpretation, but certainly if you worked or lived in the areas not suitably served by the warning siren system there would be cause for concern in an emergency situation. The potential for a coverage gap using the SCP-SO was noted as a possibility, and unfortunately it has materialized in this case. On the other hand, one would not be able to solve the associated SCP using Census blocks because $N_i = \emptyset$ for

\(^2\) The SCP-SO is an integer program. To solve this program, CPLEX relaxes integer requirements on decision variables and solves the initial problem as a linear program. Subsequent work to derive an all integer (optimal) solution requires resolving any fractional decision variables in the linear relaxation. The process for doing this in order to prove optimality is called branch and bound. Thus, iterations refers to the total number of simplex iterations executed and branches is the total number of integer assignments evaluated.
one or more blocks \( i \) in this case. Consequently, the SCP-SO now makes it possible to conduct coverage modeling in such a situation. A second issue that arises when examining Figure 5 is that there are two cases where sirens appear located in rather close proximity to each other -- in the west and in the southeast. This is a byproduct of Census block representation, where associated blocks are large relative to the service coverage area, and the fact that the model does not track whether facilities are providing complementary partial coverage. In this case, the model accounts for multiple partial coverage, but it turns out that in some cases the partial coverage is redundant rather than complementary. As a result, some blocks are not completely covered.

An alternative representation of the Dublin region was also evaluated. In this case, the SCP-SO was applied to 361 polygons uniformly delineating Dublin, most of which are 1320 x 1320 ft. in size except for those on the region boundary. The spatial pattern is similar to the tessellation shown in Figure 1b. Initially, \( \beta=100\% \) and \( \alpha=1 \) is examined. Again, this is equivalent to the SCP requiring full coverage of each polygon. In this case 31 sirens are identified as necessary, with 336.26 seconds (129,978 iterations and 2781 branches) required for problem solution. Of course, complete regional coverage is assured, but this identified number of facilities is likely conservative. In contrast, applying the SCP-SO with \( \beta=50\% \) and \( \alpha=2 \) finds that only 26 sirens are necessary for covering the entire region, requiring approximately 3 hours (2,524,304 iterations and 17,890 branches) to solve optimally. This configuration is shown in Figure 6.
Close examination of siren coverage in Figure 6 shows that a small fraction of the region is not covered. This amounts to 0.14% of the total area, which is arguably negligible. It is noteworthy that the SCP-SO is able to significantly reduce the number of necessary facilities when $\beta=50\%$ and $\alpha=2$ as compared to $\beta=100\%$ and $\alpha=1$ (the SCP case). As a result, use of the SCP suggests nearly 20% more facilities than actually necessary. A final point is that in contrast to the solution for Census blocks (Figure 5), it appears that more reasonably sized polygons relative to facility service coverage have resulted in not only a fewer number of needed facilities, but also essentially complete regional coverage.

**Discussion and Conclusions**

It is clear from the results presented that the spatial application of set covering necessarily must address notions of partial coverage if service efficiency is a primary objective. The above analysis demonstrated that ignoring partial coverage either results in instances where the set covering problem (SCP) cannot be solved due to no feasible solution existing or the number of identified facilities is either too conservative or excessive. Thus, the introduction of the set covering problem-spatial objects (SCP-SO) to account for partial coverage has enhanced capabilities for addressing coverage issues in spatial analysis. Further, the SCP-SO represents a realization of sorts on how probabilistic covering, of concern to Aly and White (1978) and Benveniste (1982), might be approached.

The application results do suggest that the SCP-SO appears to be less susceptible to the modifiable areal unit problem (MAUP). While the number of application instances is
rather small here, the number of facilities identified in the two cases when $\beta=50\%$ and $\alpha=2$ was similar (27 sirens in Figure 5 and 26 sirens in Figure 6). These are the best results (most efficient) found to date for this particular planning application (see Murray and O’Kelly 2002; Murray, O’Kelly, and Church, forthcoming).

The computational requirements for solving the SCP-SO are modest for the parameters examined here. When $\alpha>2$, however, problem structure and solvability quickly deteriorate beyond the capabilities of the software and platform utilized. For the application illustrated in Figure 6 (361 areas to cover and 4,586 potential facility sites to locate at), the SCP-SO with $\alpha=3$ and $\beta=0.35$ could not be successfully solved in 70 hours of processing time.\footnote{At the time of termination, CPLEX completed 53,525,118 iterations and 401,872 branches with an optimality gap of 10.22\% remaining. It is unlikely that any additional amount of computational processing time would be sufficient for successfully solving this particular problem using CPLEX.} In comparison to the SCP, for equivalently sized applications the SCP-SO no doubt requires greater computational effort to solve using commercial optimization software. However, the incorporation of partial coverage enables more flexibility in spatial representation, potentially allowing the SCP-SO to be applied to a smaller number of geographic units (with larger average size). Mathematically, this means fewer model constraints when considering the problem sizes reported by Murray and O’Kelly (2002) and Murray, O’Kelly, and Church (forthcoming). Nevertheless, it is clear that there is a need for research devoted to developing alternative approaches, both heuristic and exact, for solving the SCP-SO.

This paper has reviewed the use and application of set coverage in spatial analysis. While much has been done to apply and solve the SCP, its use in a geographical setting can be
problematic. The major issues are sensitivity to scale and unit definition (modifiable areal unit problem), representational inaccuracies, and computational demands for problem solution. In an effort to develop a more frame independent approach, this paper proposed the SCP-SO which enabled partial coverage to be explicitly addressed in a structured optimization model. Application results demonstrated that the SCP-SO can readily be applied in conjunction with GIS to address practical planning issues. Further, computational performance of the model was reasonable, and the SCP-SO identified the fewest number of facilities needed to serve this region (26 sirens). A geographic perspective has been responsible for rethinking how service coverage is modeled, and how partial coverage could be addressed in a quantitative framework.

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Figure 1. Regional abstraction.

Figure 2. Warning siren service coverage (70 dB range).

Figure 3. Polygon coverage.

Figure 4. Non-complementary coverage.

Figure 5. SCP-SO siting configuration for covering Dublin Census blocks.

Figure 6. SCP-SO siting configuration for covering regular tessellations of Dublin (1320x1320 ft.).
(a) Blocks and block centroids

(b) Regularly spaced tessellation